BUCKLING OF STRUCTURAL MEMBERS

BY

ALPHONSE ANTHONY BRIELMAIER

B.S., Case School of Applied Science, 1926
C.E., Case School of Applied Science, 1929
M.S., University of Illinois, 1930

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1948

URBANA, ILLINOIS
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This thesis was begun in 1937 while the writer was employed on a full-time teaching appointment at the University of Illinois. The writer was able to devote part of the time during the academic years from 1937 to 1941 and the summers of 1938, 1939, and 1940 to working on the thesis. The main body of the thesis was completed by June, 1941, when the writer left the University to engage in the design of war plants with the United States Corps of Engineers.

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The writer is grateful to Professor Nathan M. Newmark of the University of Illinois for first interesting him in the general subject of buckling and for helpful comments and suggestions. The stimulating discussions of various phases of this thesis with Thomas C. Shedd, Professor of Structural Engineering at the University of Illinois, are deeply appreciated. The writer also wishes to take this opportunity to acknowledge the inspiration gained from Professor Hardy Cross, now of Yale University, through attendance in his graduate courses. The writer is indebted to Drs. J. A. Sauer, D. J. Peery, and J. Marin for helpful discussions.

In this thesis, a determination is made of the deflected shape and ultimate or "buckling" load of a number of columns — varying in shape, loading, and end conditions — by a method of analysis that consists essentially of successive approximations.

This method of analysis requires the assumption of a deflected shape of the loaded column, the computation of moments due to this deflection and to the loads, and the computation by means of the moment-area principle and the conjugate-beam concept, of the deflections corresponding to the computed moments. If the computed moments do not agree with those assumed, a second trial solution is made, beginning with the computed deflection curve. The trials are continued until satisfactory agreement is found. For the column with axial load only, the load that will just maintain the column in its position of slight deflection is the critical or "buckling" load, since any increase in this load will result in increasingly greater deflections leading to the failure of the column.

The method of analysis just described — while it may sometimes require more laborious computations than the more mathematical approach — is usable for cases in which the mathematical approach is unworkable.[1] It has the additional advantage of making use of the moment-area principle which is familiar to most structural designers. The first presentation of the method is usually credited to Luigi Vianello[2] in 1898. Other writers who have published articles based on the same approach are Land[3] (1896), G. Duclout[4] (1896), A. Morley[5] (1917), and N. M. Newmark[6] (1942). The present writer, however, is not aware of any work in which this method has been systematically applied to a variety of problems in column buckling, with a view to studying its adaptability and also its accuracy where the results of a rigorous mathematical solution are available. It is intended in this thesis to make such application and where possible, such a comparison of results.

[1] Timoshenko, S., *Theory of Elastic Stability.* (New York: McGraw-Hill Book Co., Inc., 1936), pp. 114, 130.

[2] Vianello, L., "Graphische Untersuchung der Knickfestigkeit gerader Stäbe," *Zeitschrift des Vereines deutscher Ingenieure,* Vol. 42 (1898).

[3] Land, R., "Einfache Ableitung der Eulerschen Knickformel," *Zeitschrift des Vereines deutscher Ingenieure,* Vol. 30 (1896).

[4] Duclout, C., "Théorie du Flambage des Pièces Élastiques Comprimées," *Mem. Soc. Ing. Civ. de France,* September, 1896.

[5] Morley, A., "Critical Loads of Long Tapering Struts," *Engineering,* September 21, 1917.

[6] Newmark, N. M., "Numerical Procedure for Computing Deflections, Moments and Buckling Loads," *Proc. Am. Soc. Civil Eng.,* May, 1942.

The problems considered include columns with constant moment of inertia and with moments of inertia varying gradually and varying abruptly, with ends that are hinged, fixed, or unrestrained; with end loads that are axial, or eccentric but parallel to the axis; with axial loads that are applied at an intermediate point along the axis or are distributed along the axis; and with transverse loads acting in combination with axial loads. Several specific numerical problems are treated, including a rolled steel column reinforced with cover plates over part of its length, a derrick boom, and a suspension bridge tower column.

Four standard cases of the column of uniform cross-section, axially loaded at the ends and with various end conditions are analyzed. The deflected shape and the buckling load are determined with both ends hinged, one end hinged and one end fixed, one end free. For the first case, five trial solutions were made which resulted in values of the buckling load differing from the correct value by 3.4 per cent, 1.2 per cent, 0.8 per cent, 0.3 per cent, and 0.2 per cent respectively. For the second case, three trial solutions were made which yielded buckling loads that were in error after the first trial by less than 1 per cent. There were two trial solutions for the third case, the column with both ends fixed. The first resulted in a buckling load which was 1.0 per cent below the correct value and the second resulted in a buckling load which was 0.75 per cent below the correct value. The buckling load for the fourth case was obtained from the analysis of the first case. The buckling load, based on the third trial solution, is 0.8 per cent below the correct value.

Two groups of columns axially loaded at hinged ends and with moment of inertia varying symmetrically about the midlength, are studied for the shape of the deflection curve and for buckling load. In the first group, twenty-four variations of column shape are assumed in all of which the moment of inertia increases gradually from the ends toward the midlength. In the second group, sixteen variations of column shape are assumed in all of which the moment of inertia is constant over the outer portions and of a greater but also constant value over the center portion of the length. The results of a rigorous mathematical solution are available[7] for some of these columns. A comparison of buckling constants indicates satisfactory agreement. The results of the analyses are given in a graphical form, usable in design. The shapes of the deflection curves are also given in plotted form.

In addition to the study of columns with various end conditions and of various shapes, the method of analysis is also applied to columns

[7] Timoshenko, S., *op. cit.*; Ratzersdorfer, J., *Die Knickfestigkeit von Stäben und Stabwerken.* (Vienna: Verlag von Julius Springer, 1936).

with various loading conditions. The columns considered are assumed to have constant moment of inertia and hinged ends and to be subjected to the following loadings: axial load at ends and uniform lateral load; axial load at ends and lateral load concentrated at midlength; eccentric load at ends, parallel to axis of unloaded column; and eccentric load at ends, parallel to axis of unloaded column and uniform lateral load. A general analysis is made for these four cases by means of an infinite series describing the deflection at midlength of the column. An approximation is involved in obtaining the sum of the series. The results of the analyses for the first and third cases are presented in algebraic and in graphical form, suitable for design use.

An analysis is also made of several columns of constant moment of inertia, with various end conditions but with axial loads not limited to the ends. These cases consist of a column hinged at both ends and with uniform axial load; a column fixed at one end and free at the other, with uniform axial load; and a column hinged at both ends, with an axial load at the ends and at midlength. The shape of deflection curve and the value of the buckling load are obtained for the first two cases. A general solution is made for the third case, yielding an expression for the buckling load for any ratio of load at top of column to total load. The deviations of the obtained buckling load from the correct buckling load, for load ratios of 1.0, 0.8 and 0.5 are 0.9 per cent, 1.1 per cent, and 1.0 per cent respectively. The unit stress at failure by buckling is plotted against the slenderness ratio for load ratios of 1.0, 0.5, and 0. It may be noted that when the entire load is at midlength, the buckling load is 91 per cent greater than when the entire load is applied at the top of the column.

A general solution is also given of a column similar to the above except that the moment of inertia while constant throughout each half-length, changes abruptly at midlength and is greater in the lower half. For several relative values of the moments of inertia in the two halves of the column, an expression is derived between the buckling constant and the ratio of top load to total load. The buckling constants computed for four representative sets of ratios of moments of inertia and of top load to total load are compared with values obtained by the rigorous approach. The differences varying from 1 per cent to 3 per cent are somewhat high because the general solution in this case is based on only one assumed deflection curve.

The method of analysis is applied specifically to four numerical problems involving a wide-flange rolled steel member, hinged at both ends. In the first problem the column is axially loaded at the ends, and is designed as a member of constant section and then as a member

of constant section reinforced with two cover plates over the middle portion of its length. It is found that the latter design requires 14 per cent less weight of steel. The second problem concerns a column of constant section axially loaded and with a lateral load concentrated at midlength. Several curves are drawn, showing the variation of the ratio of yield point to maximum extreme fiber stress, with the sum of the ratios of actual to allowable stresses for the member as a column and for the member as a beam. The curves represent several values of the axial and lateral loads. The third problem is a column of constant section, eccentrically loaded at the ends. Various values are assumed for the direct load and for each value several values are assumed for the eccentricity. The deflections, maximum stresses, and ratios of buckling load to direct load, yield point to maximum stress, and actual stress to allowable stress are computed. The results are shown graphically. The fourth problem deals with a column axially loaded at the top and at midlength. Four designs are made for the column; as a member of constant section with the entire load at the ends; as a member of constant section with the loads at the ends and at midlength; as a member of constant section reinforced with cover plates over the lower half-length (two designs). The total weight of steel required for each of the four designs is 9,690, 6,820, 7,400, and 6,450 pounds respectively.

A derrick boom, 95 feet long, built up of angles, plates, and lacing, and with its moment of inertia varying almost symmetrically about midlength, is the subject of a somewhat extended study. The study includes the determination of the buckling load, the computation of deflections for axial end load combined with uniform lateral load, for eccentric end loads, and for eccentric end loads combined with uniform lateral load. The allowable hook load on the derrick, with a factor of safety of four, is determined for the boom in a horizontal position. If the lacing near the boom ends is replaced by side plates several feet long and the pins are located four inches below the boom axis, it is found that the hook load may be increased by 22 per cent, for the boom in the horizontal position only.

In another numerical problem, the tower column of a suspension bridge is investigated. It is first considered in a general manner by the study of four loading cases and then specifically by a more detailed study of one loading case. The general cases include the determination of the buckling load for the column fixed at the base and hinged at the top and also for the column fixed at the base and free at the top; the computation of deflections for axial and lateral loads at the top with the top free, and for eccentric loads with the top free. The specific loading case includes consideration of the cable reactions and stiffening

truss reactions due to dead load and live load, of the weight of the column and bracing considered as a load distributed along the column, and of a rise in temperature. In this case, the movement of the top of the tower is governed mainly by the change in length of cable due to stress and temperature rise. The deflections along the column, and the extreme fiber stresses are calculated and plotted.

The application of this method of successive approximations, based on the moment-area principle, to a variety of cases illustrates its ready adaptability to problems of interest to the structural engineer. The satisfactory accuracy of the results is indicated both by comparison with available results obtained by the rigorous mathematical analysis and also by the comparison of assumed and computed deflection curves.

VITA

The writer of this thesis was born in Chicago, Illinois, on March 23, 1905. He graduated from Loyola High School in Cleveland, Ohio, in 1921. From 1922 to 1926 he studied civil engineering at Case School of Applied Science and received the degree of B.S. The next three years were spent as a structural detailer and designer, mainly on reinforced concrete buildings, with The Whitacre Engineering Company and as office engineer with The Boldt Construction Company, general building contractors, both in Cleveland, Ohio. In 1929 he received the C.E. degree from Case School of Applied Science. From 1929 to 1930 he was a graduate student in structural engineering at the University of Illinois, and was granted the M.S. degree. The following summer he spent as a structural designer with the consulting firm of Lord & Holinger in Chicago. During the academic year 1930-31 he was an International Exchange Fellow at the Technische Hochschule in Danzig.

In 1934, after one and one-half years as engineer and superintendent in soil erosion camps under the Emergency Conservation Work program, he was employed by the United States Engineer Department on the design of hydraulic structures and bridges in Zanesville, Ohio, and Los Angeles, California. He returned to the University of Illinois in 1936 for further graduate study. During the period 1937-41, he taught courses in structural engineering at the University of Illinois. From 1941 to 1945 he was with the United States Engineer Department in Denison, Texas, and at Omaha, Nebraska. At Denison, he was responsible for checking the structural design of the Fort Worth Aircraft Assembly Plant and for the structural design of the powerhouse at the Denison Dam. At Omaha he was employed on preliminary designs for the powerhouse and chute spillway at the Garrison Dam on the Missouri River in North Dakota.

Since 1945 he has been associate professor of civil engineering at The Pennsylvania State College, State College, Pennsylvania, teaching undergraduate and graduate courses in structural engineering. During the summer of 1945 he was in Europe with the Army Air Forces, studying German research data, and during the summer of 1946 he was with the firm of Freese & Nichols, consulting engineers of Dallas, Texas, where he was responsible for the design of a series of continuous girder highway bridges.

He is a member of the American Society of Civil Engineers (Junior 1928, Associate Member 1935, Member 1947), the American Concrete Institute, The National Society of Professional Engineers, and Sigma Xi. He is registered as a structural engineer in Illinois and as a professional engineer in Illinois and Pennsylvania.